

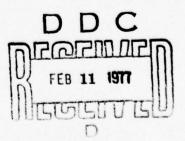
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SHOCK WAVE PENETRATION AND LATERAL PRESSURE GRADIENT EFFECTS ON TRANSONIC NORMAL SHOCK-TURBULENT BOUNDARY LAYER INTERACTIONS+

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NOMENCLATURE

- C. local skin friction
- $h_i P_i'/P_{0i} (i = 1, 2, 3)$
- H_i Fourier transform of h_i
- k wave number in Fourier Transform
- £ sublayer thickness (see → Eq. (6)
- p static pressure
- Q unit function defined in Eq. (4)
- Re Reynolds number based on L
- u. v x- and y- velocity components
- x, y coordinates parallel and perpendicular to freestream
- $(M^2 1)^{1/2}$ or $(1 M^2)^{1/2}$
- γ ratio of specific heats
- ⁶BL boundary layer thickness
- ΔP basic static pressure jump across shock
- n displaced interface location
- H Viscosity coefficient

- * Fourier Transform of pressure disturbance function
- o density
- exponent for power law viscosity law $(\mu \sim T^{\omega})$

Superscripts

()' disturbance quantity

Subscripts

- e freestream properties upstream of shock at boundary layer edge
- o denotes undisturbed (not stagnation) flow property
- L length to undisturbed shock location
- w property at wall value
- i = 1, 2, 3, denotes region of variable
- LS based on Lighthill sublayer

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ABSTRACT

The titled affects, which are usually neglected in contemporary theories of viscous-inviscid interaction, are here examined in detail for the unseparated transonic turbulent case. They are found to be significant in many respects. In particular, our results suggest that neglect of shock penetration becomes very inaccurate in the supersonic separated flow regime, leading to substantial underestimates of the local interact strength and its overall streamwise extent.

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INTRODUCTION

Basic studies of shock wave-turbulent boundary layer interaction find significant practical application in various aerodynamic and propulsion system flow problems. In existing theories the impinging shock is imposed as a outer disturbance boundary condition at the boundary layer edge but the local details of its subsequent penetration into the layer neglected in obtaining the interactive flow field solution. Moreover, the attendant simplification of neglecting the lateral interaction-pressure gradient across the boundary layer is often made. These approximations are usually justified a priori on the grounds that the resulting errors in the important large scale features of the flow are insignificant for engineering purposes. Indeed, up to moderate supersonic speeds this has been found true in laminar flows because of their well-spread out response to even weak shocks and the small attendent lateral pressure variations across the boundary layer. However, for turbulent flows at finite Reynolds numbers of practical interest (10⁶ ≤ Re_L ≤ 10⁸) the interaction is much more violent and short range 1 (± 2 - 4 δ) and

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hence the shock penetration and lateral pressure gradient effects may become important. Werle and Bertke have recently given strong evidence that this is the case: detailed comparisons show that their interacting supersonic boundary layer model solutions (which have otherwise given consistantly good results in laminar flows) severely misrepresent both experiental data and exact Navier-Stokes solutions for a M_{∞} = 3 separating Re_{L} = 10^{7} turbulent flow regardless of the eddy viscosity model employed. They concluded that the failure of the model is due to its lack of account for the shock penetration into the boundary layer; moreover, neglected lateral pressure gradient terms may also play a significant role in such cases.

The present paper deals with these aspects of the interaction problem for the case of transonic normal shocks interacting with non-separating turbulent boundary layers. Although acknowledged to be in a lower speed range without separation the results nevertheless shed valuable light on the nature and parametric dependence of the shock penetration and $\delta p/\delta y$ effects which proves useful in other situations. The approach is based on an extension of a recently-developed approximate theory 4 of this transonic interaction problem.

THEORETICAL MODEL

The flow is taken to consist of a known unseparated turbulent boundary layer profile M_0 (y) subjected to small transonic disturbances due to an impinging weak normal shock ($M_1 < 1.3$). Our original theoretical model of this interaction was an approximate simplification of the disturbance flow structure emerging from an asymptotic analysis of the compressible Navier-Stokes equations at high Reynolds numbers, giving a linearized boundary value problem surrounding the nonlinear shock discontinuity and underlaid by a thin Lighthill viscous disturbance sublayer (Fig. 1). Provided the incident shock is weak enough to avoid a separated lambda-shock interaction pattern while M_1 is not so close to unity that the viscous structure within the incident shock must be considered, this flow model represents all the essential

global features of the mixed transonic character of the non-separating normal shock-boundary layer interaction problem 6 including lateral pressure gradient effects. Moreover, the linearized theory involved is amenable to analytical treatment by obtaining solutions in each of the three regions shown in Fig. 1 and matching them along the boundary layer edge interface by continuity of pressure and streamline shlope. The details, carried out by operational methods, are given elsewhere 4 . The results include interaction pressure and streamline deflection along the boundary layer edge and both the wall pressure and shear distributions over a wide range of $\rm M_1$ and $\rm Re_L$ values for an arbitrary incoming turbulent boundary layer profile, whether adiabatic or not 7 .

As orignally formulated, the aforementioned theory not only assumes that the incident shock and its extension down to the sonic line are simple Rankine-Hugoniot discontinuities, but also neglects (following Lighthills treatment of oblique shock interaction) the detailed shock penetration into the boundary layer by imposing the shock jump conditions only at the outer edge; since the correct shock pressure jump at the edge is accounted for while below the sonic level in the boundary layer no discontinuity can exist, the shock decay across the supersonic non-uniform flow region is in fact roughly simulated by this approximation. The ultimate result is a small wall pressure discontinuity under the shock (Fig. 2) whose effect can be easily eliminated by an equivalent continuous redistribution of the jump over the entire interaction. Nevertheless, since it is of fundamental and practical interest to improve on this approximation, a modification of the theory to account for the shock penetration effect was sought.

An earlier preliminary study has shown that except very near the sonic level y_s , the shock in a turbulent boundary layer remains essentially normal and simply decays in strength inward as the local Mach number drops. But since the penetration region $\delta_0 \le y \le y_s$ lies in the outer "velocity-defect" portion of the turbulent profile where the Mach number gradient is very weak, this strength decay is also small except when $y \longrightarrow y_s$ (see Fig. 3). For practical high Reynolds number interactions, these facts suggest that a first approximation (and indeed an upper limit) to the shock penetration

effect can be obtained by continuing the full incident shock unchanged across the supersonic region. However, to the same order of approximation, the effect of this is equivalent to a "non-penetrated" interaction solution for an incoming boundary layer having the same skin friction (wall slope) but a smaller thickness $\delta_0' = y_s$. In other words the shock penetration effect in the leading approximation is simply equivalent to a distortion (primarily in thickness) of the boundary layer profile. This idea is easily applied by running the existing program twice: the first run establishes δ_0 , $\eta_s = y_s/\delta_0$, C_{f_0} , etc. and the associated "unpenetrated" interaction field solution, while in the second the programmed equation for δ_0 is multiplied by η_s (as are all values of Re_{δ} where ever they appear in calculating the Mach number profile, Lighthill sublayer properties, etc.) taking care not to change C_{f_0} ; the results give the interaction field with shock penetration.

DISCUSSION OF RESULTS

Lateral Pressure Gradient Effects

Typical interaction solutions are illustrated in Fig. 4, where both the boundary layer edge and wall pressure distributions are shown. It is seen that the shock-induced lateral pressure gradients are significant within a region of several boundary layer thicknesses upstream and downstream of the shock foot, the wall pressure being higher then the edge ahead of the shock and lower behind it. Further upstream where the wall and edge pressure equalize they decay exponentially with distance. Along the boundary layer edge, a local pressure jump occurs across the shock at x = 0 followed by a small region of subsonic post-shock expansion and subsequent recompres, sion, in qualitative agreement with experimental observations x = 0 followed by a downstream, the wall and edge pressures again equalize and rise monotonically to the final post-shock level like x = 1

It is emphasized that the local shock jump at the boundary layer edge and its rapid lateral smoothing across the underlying subsonic flow region that yields a continuous wall pressure distribution are important physical

features that cannot be accounted for without the lateral pressure gradient effect. Indeed, consideration of the former provides one means of checking against other theories and experiment; for example Fig. 5 compares the predicted local interaction-pressure jump as a function of shock strength against a variety of observed values 11 . Within the validity of the theory (unseparated flow, $\rm M_1 < 1.3$) our results agree well with the data and show the qualitatively-correct trend of approaching the full inviscid Rankine-Hugoniot result with increasing $\rm M_1$ or Reynolds number.

The inclusion of lateral pressure gradients captures another interesting feature, the existence of a subsonic post-shock expansion region at the boundary layer edge (here due to sign change across the normal shock ¹² of the upstream compression waves from the interaction-induced boundary layer thickening, see Fig. 1). Not only is this qualitatively confirmed by experiment and detailed numerical solutions ¹³ but also its behaviour with respect to Mach number: as predicted in Fig. 4, Ackeret, Feldman and Rott ⁵ have observed that the degree of expansion and the extent of its influence into the boundary layer gradually disappear with increasing Mach number. Moreover, this phenomena plus the aformentioned rapid lateral pressure variations are clearly evident in their detailed pressure surveys of the interaction region (see Fig. 6): outside the boundary layer there is a noticeable overshoot immediately behind the shock of the magnitude shown in Fig. 4.

The influence of Reynolds number on the lateral pressure gradient effect is also significant as shown in Fig. 7. The local shock jump and post-shock expansion increase and decrease, respectively, with increasing Re and at sufficiently high values the post-shock expansion region is very small and weak and hence difficult to detect experimentally. An account of this expansion feature is of practical interest not only in understanding the physics of the flow but also in sorting out various theories. For example, mixed transonic flow past a curved wall—exhibits post shock pressure dip but for a completely different reason (e.g., as a result of the purely inviscid disturbance flow logarithmic singularity associated with convex surface curvature 14.) To properly differentiate between this effect and other influence requires consideration of $\delta p/\delta y$.

Shock Penetration Effects

The effect on the interaction pressure field as calculated from our approximate model is shown in Fig. 8; the corresponding boundary layer thickening and skin friction are shown in Figs. 9 and 10. It is seen that for the weaker shock cases (M₁ < 1.15) the interaction contracts and thins out slightly with increased skin friction, whereas for stronger shocks the reverse becomes true and shock penetration strengthens the interaction by spreading out the streamwise scale, thickening the boundary layer and reducing the local skin friction. The reason for this trend reversal is that shock penetration introduces two effects of opposite sign: for weak shocks the added shock segment acts as an equivalently stronger shock at a slightly higher Mach number, which tends to reduce the spread and strength of the interaction, whereas at higher shock Mach numbers it strengthens the interaction through increased upstream influence and thickening plus hastening of incipient separation, these rapidly becoming the dominant effects.

A parametric study of Reynolds number influence was also carried out; the results are summarized in Figures 11 - 14 showing respectively the relative changes in non-dimensional upstream influence distance (here defined as where $p^{\tau}=0.05~\Delta P)$, wall pressure, skin friction and displacement thickness growth at the shock foot. These curves further illustrate the intensification of the interaction that occurs with increasing shock strength and also its sensitivity to Reynolds number when $Re_L < 10^8$. For example, while the upstream influence of a M_1 = 1.20 shock is increased only 6% by the shock penetration effect at Re_L = 10^6 , this increase becomes 34% at only a slightly lower value Re_L = 3 x 10^6 (the corresponding thickening under the shock goes from negligable to 40%).

Although the present analysis pertains to transonic unseparated flow, its parametric trends clearly predict that the shock penetration effect increases with both shock strength, Mach number and the onset of separation (equivalent qualitatively to a lower ${\rm Re}_{\rm L}$). This is borne out by Werle and Bertke's finding (reproduced here in Fig. 15) that omission of this effect in the turbulent case significantly underestimates the strength of the interaction

and its streamwise extent. The present results support their contention that shock penetration is an important feature of interacting high speed turbulent boundary layers. Moreover, to treat it properly the lateral pressure variations must also be included.

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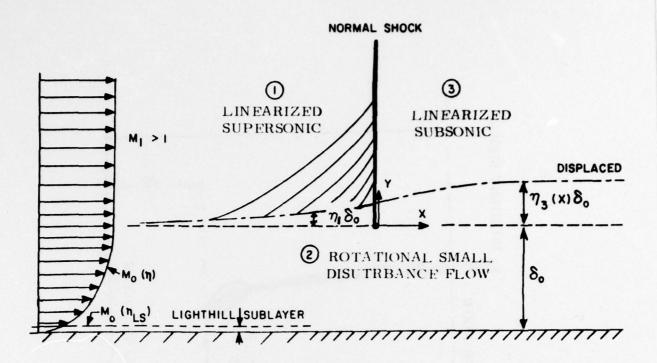


Fig. 1. Basic Interaction flow Model

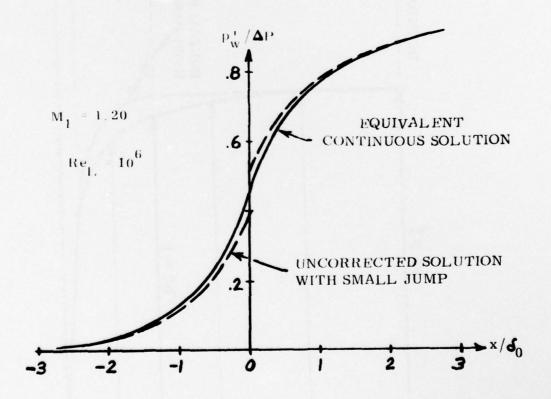


Fig. 2 Wall Pressure Solution Neglecting Shock Penetration Effect

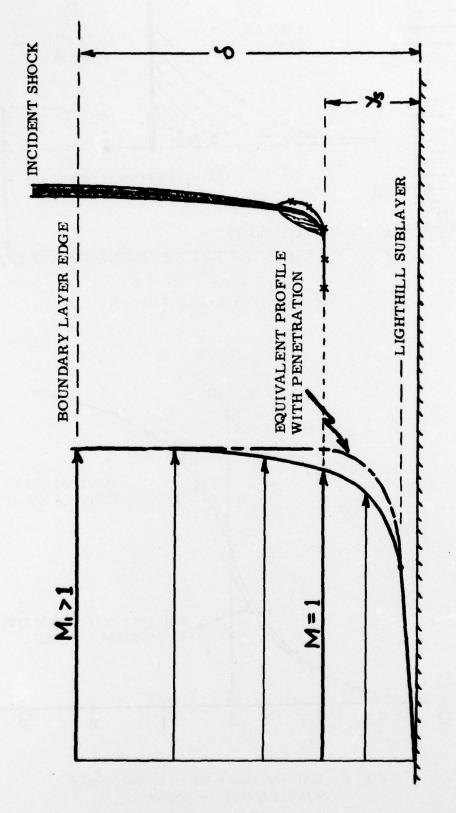
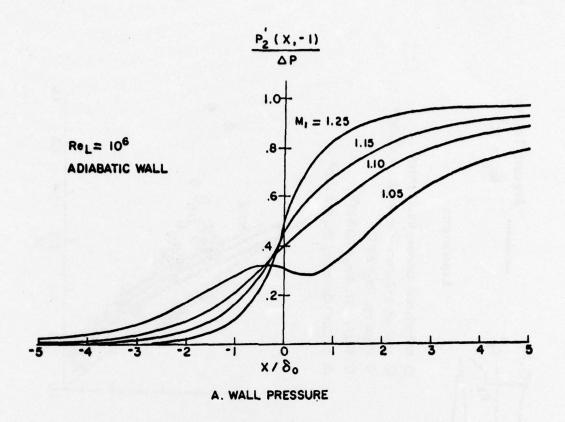


Fig. 3 Shock Penetration Region (Schematic)



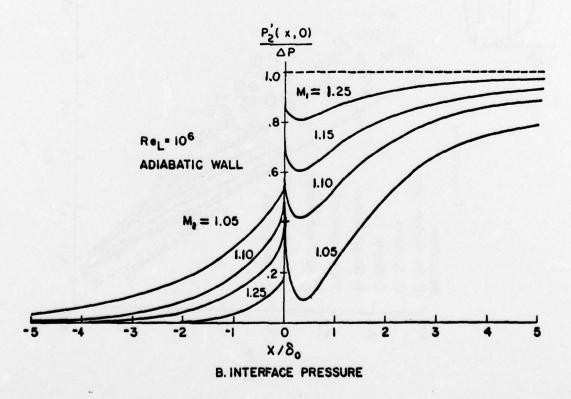


Fig. 4 Influence of Shock Strength on Interaction Pressure Field

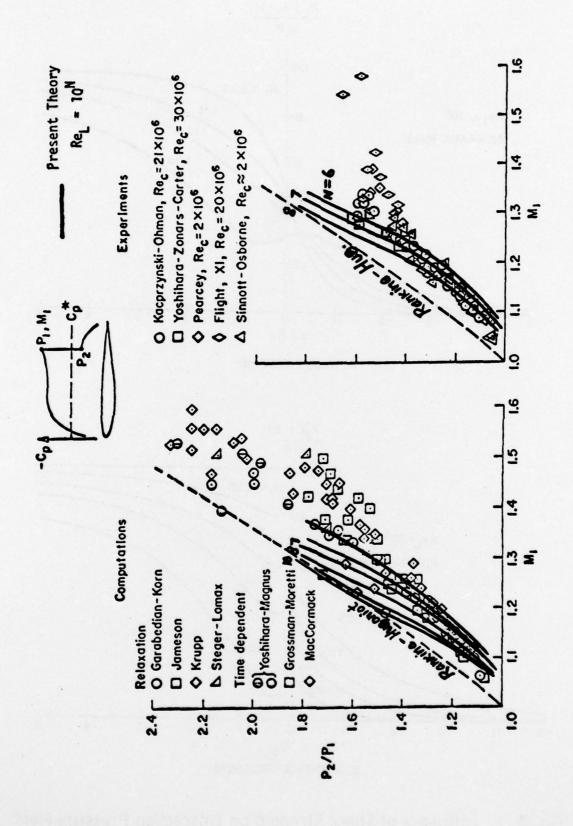
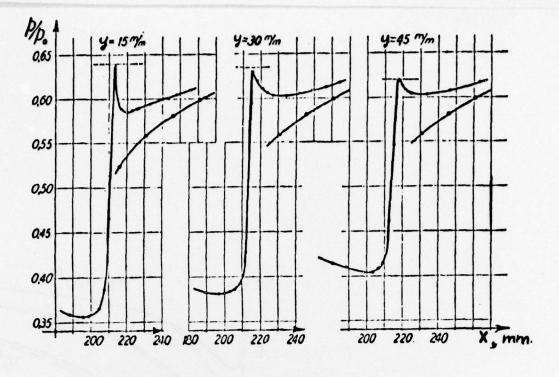


Fig. **5** Comparison of Theory and Experiment for Local Shock Wave Pressure Jump



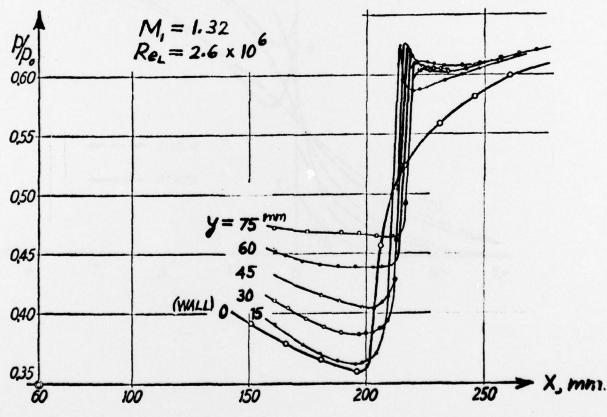


Fig. 6 Typical Experimentally Observed Lateral Static Pressure Variations Across Interaction Region 5

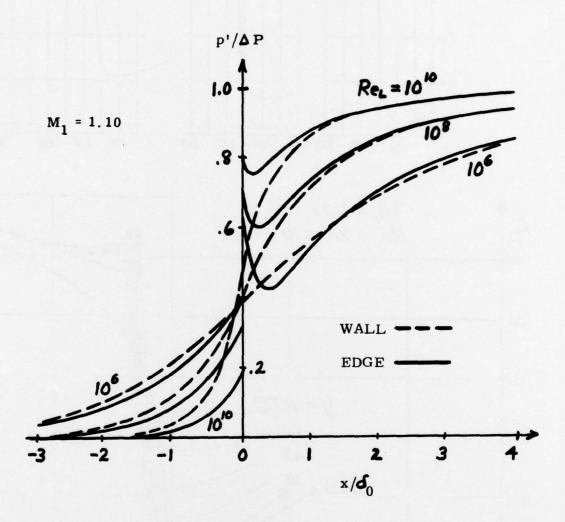


Fig. 7 Reynolds No. Effect on Interaction Pressure Field Across Boundary Layer

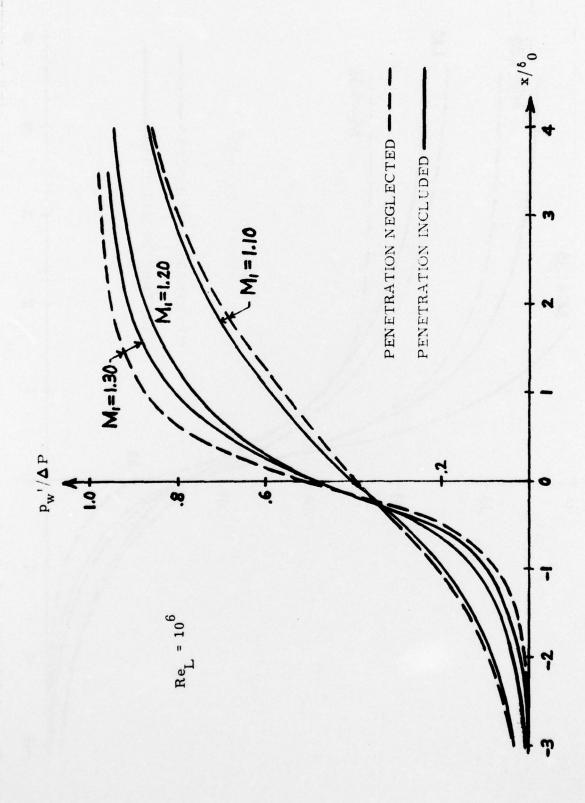
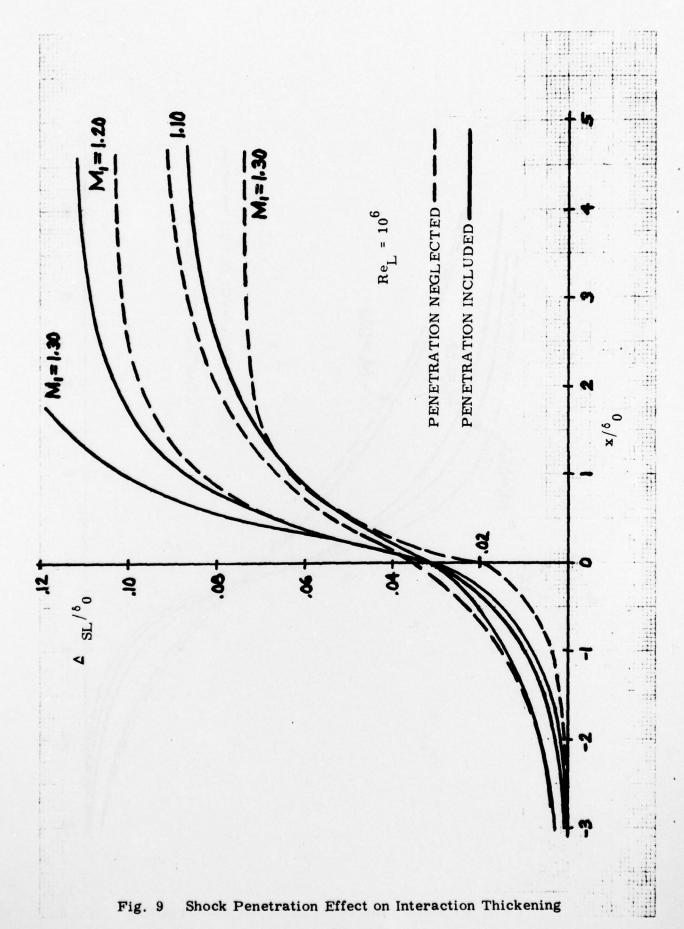


Fig. 8 Shock Penetration Effect on Wall Pressure



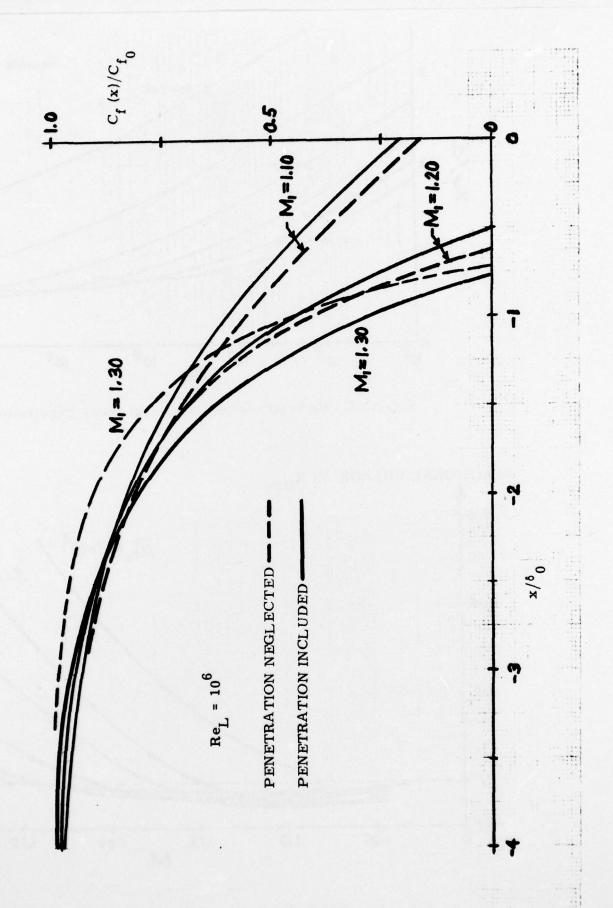


Fig. 10 Shock Penetration Effect on Skin Friction

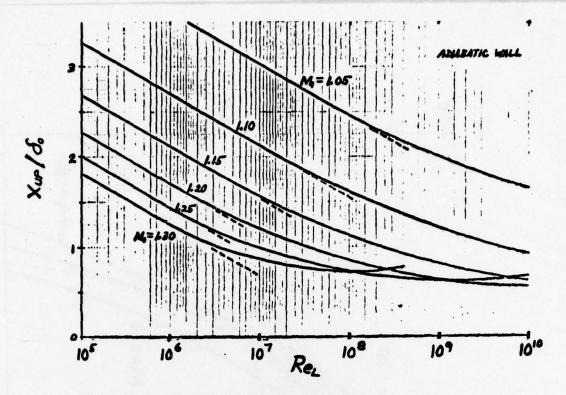


Fig. 11 A. Upstream Influence without shock Penetration

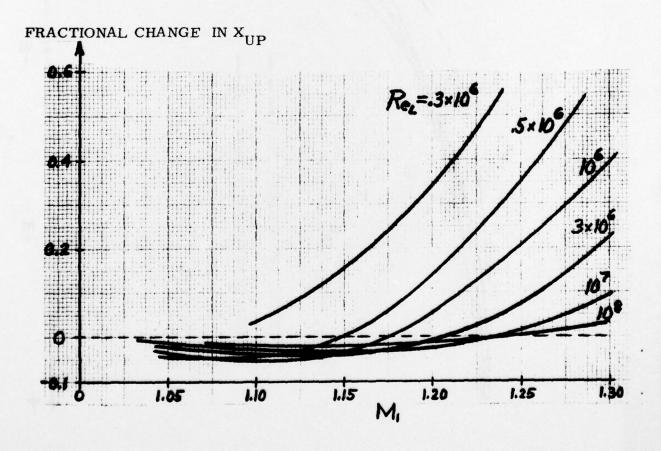
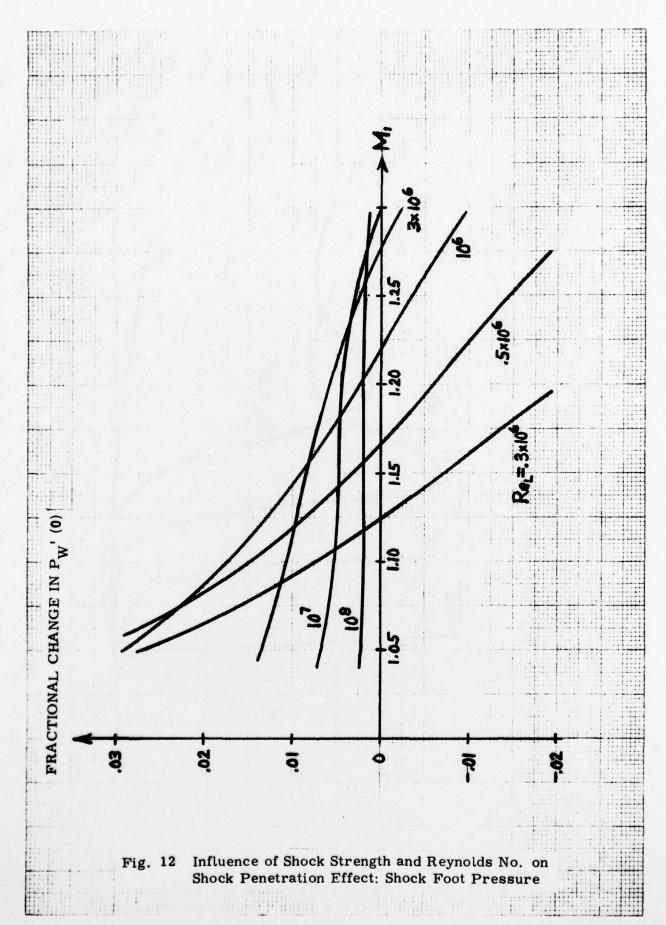
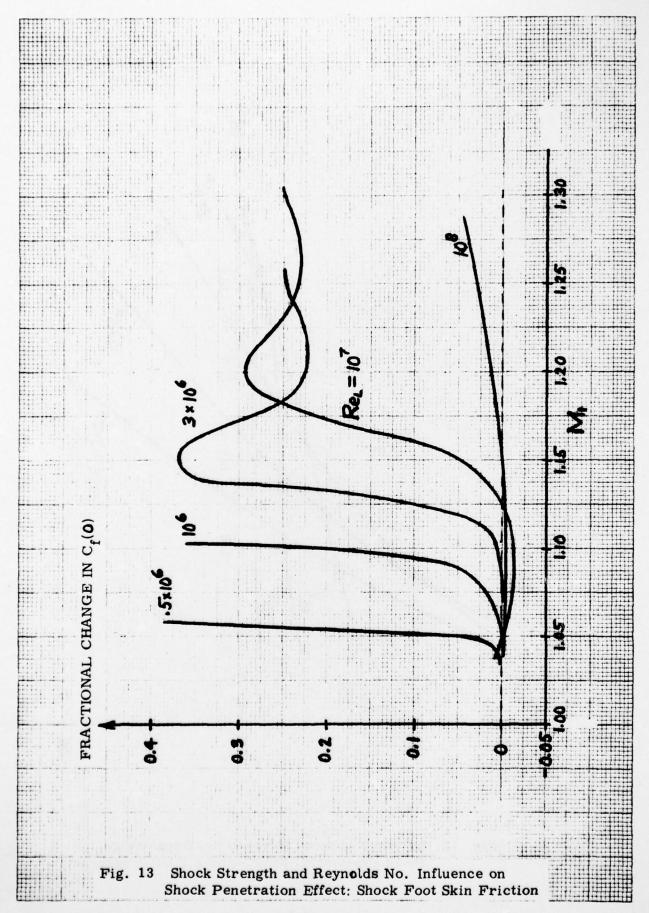
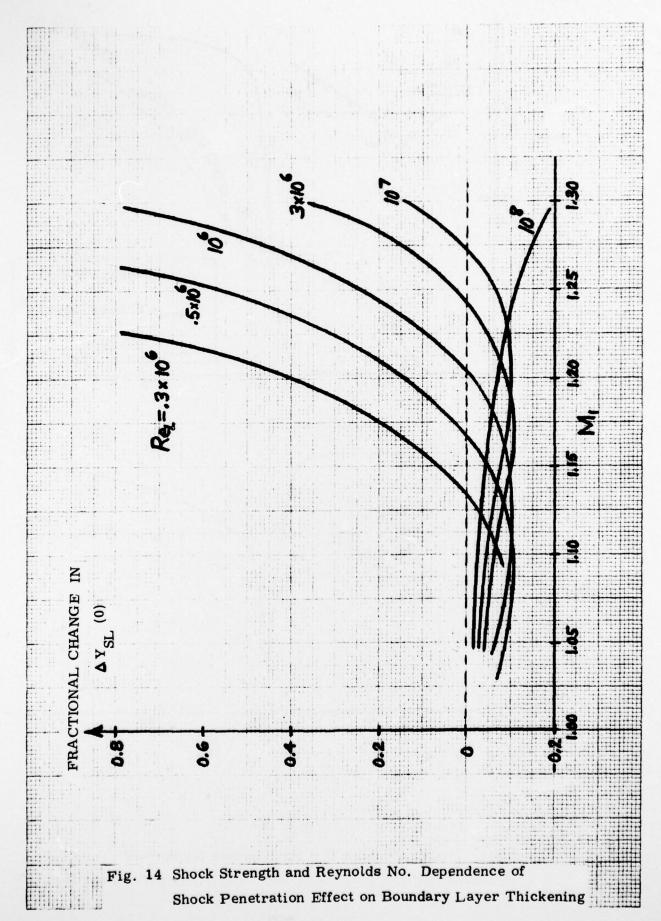


Fig. 11B Shock Penetration Effect on Upstream Influence







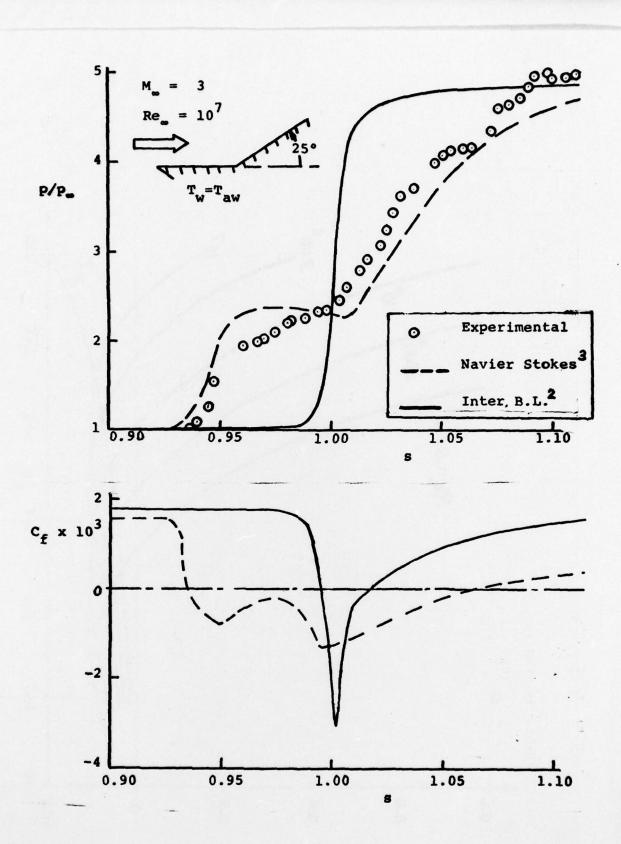


Fig. 15

Comparison of Theories and Experiments for Surface

Pressure and Skin Friction on ARL Ramp (from Werle and Bertke)

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